

Two scenarios of the evolution of modern pension systems*

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In 1958, Paul Samuelson published a paper entitled “An exact consumption-loan model of interest with or without the social contrivance of money” (Samuelson, 1958), which is widely recognised as having laid the theoretical foundations for pension economics. It is seen as providing a sort of theoretical groundwork for the scheme referred to as the pay-as-you-go pension system. Samuelson asserts that it is impossible to find an optimal solution for old-age consumption on a market basis in an economy without money and opportunities to set aside reserves, and that a Hobbes-Rousseau style of social contract is therefore required (as embodied in the modern system of social insurance institutions), according to which active generations support the older generation that they follow and, likewise, will be supported by the next generations, who are in childhood or have not even yet been born. In his model, Samuelson does not talk about child-rearing; he does not even include childhood in his model. He puts the cost of raising a child at zero, even though he admits that a sufficient number of children is required to maintain modern social insurance. This paper tackles the question of what pension system will result if Samuelson’s simplified assumption about children is removed and his model is extended to include childhood. The outcome is surprising: on basically the same foundations, a completely different scenario can be built, which would result in a system that is very similar to the present-day pension system but deviates from it in one or two significant ways. These two scenarios – this new system and Samuelson’s, respectively – are here called the “IAI” (inactive-active-inactive) scenario and the “AI” (inactive-active) scenario; the investigation focuses on how the pension systems predicated on these two scenarios resemble each other and where they differ. The most important element in the IAI scenario, which can be considered a modernised version of what Samuelson calls the “traditional pension system” (where children were the means of old-age support) that went out of fashion, is that the pension contribution should not be considered here as a sort of investment, giving entitlement to subsequent pension, as Samuelson does in his AI scenario. Instead it is treated as repayment, organised by the state and due to parents for raising children; similarly to the “traditional pension system”, this secures their sustenance in old age (i.e. their pension). This is also a social contract;

* The views expressed in this article are those of the author(s) and do not necessarily reflect the official view of the Magyar Nemzeti Bank.
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however, only parents who have raised children will be eligible for a pension under this contract. Those who do not do so will save on the costs of childcare, and the state must organise for them another type of pension system, a funded one in which the childcare costs saved are put aside and from which their pension is drawn. The conclusion of this paper includes a review of approaches published to date in the literature, which treat the problem of pension and child-rearing in ways differently than here.

JEL codes: B22, D30, H55

Keywords: Samuelson, old-age pension, pension system, child-rearing

1 Introduction

Paul Samuelson's 1958 paper (Samuelson, 1958) creates a sort of theoretical groundwork for the so-called pay-as-you-go (PAYG) system, which is widespread in the developed world and the long-term sustainability of which is being questioned almost everywhere.

This paper investigates what pension system emerges if some of Samuelson's original simplifications are resolved and the events are interpreted differently at certain points. The result is surprising: a wholly different scenario can be constructed on essentially the same foundations.

The following section briefly examines the main points in Samuelson's original paper and then draws attention to one of his abstractions, which is subsequently resolved.

It can be seen that Samuelson's model is not robust when it comes to results: the resolution of the minor abstraction results in a radically different picture of pensions. The two scenarios – this one and Samuelson's – represent entirely different pictures; even though built on similar grounds, they lead to completely different conclusions at key points.

2 Samuelson's original article and its abstractions

In his paper, Samuelson seeks to answer the question of whether, in an economy shared by three generations of people and in which the consumer goods produced will not keep over the long term, a free-market exchange mechanism is at play to ensure that the older, currently active persons share some of their output with younger active persons in order to make sure that the latter will share theirs with them later on, so they do not die of

hunger in old age. For the sake of simplification, he assumes that in this “economy” only one type of goods is produced and consumed: perishable chocolate. He recognises three generations: young, employed persons constitute the first generation; older, employed persons comprise the second generation; and inactive persons, who no longer work, make up the third generation. He does not deal with children specifically; they exist by default and do not represent substantial consumption. Accordingly, Samuelson disregards them as a factor (and includes their consumption within that of the parents); only active and retired persons have significant consumption.

In order to analyse the issue, Samuelson posits a simple model with several simplifications (some of which have already been mentioned) and formulates his main question in the following way: what interest rate will lead to equilibrium and social optimum (i.e. a maximum value in the utility function analysed)? He finds that a 0% rate of interest would lead to a socially optimal level in a stationary population, which he demonstrates by means of a mathematical example: if everyone produces 1 unit per period (that is to say, a total of 2 units over their entire life), then they will consume $\frac{2}{3}$ in each period (i.e. in their old age, they will receive $\frac{1}{3}$ each from two persons, who are in their active periods). This, however, is irreconcilable with the free-market mechanism, as it is unimaginable that older, active persons (in their 2nd period) would hand over some of the goods they have produced to bribe active persons in their 1st period to ensure that the latter will share with them some of the goods that they produce in their 2nd period, once the former are pensioners in their 3rd period. On a free-market basis, this either does not add up or only does so at a horrendous rate of interest. This means that those in their 2nd period would need to give a huge amount to those in their 1st period if they wished to receive a little back in their 3rd period from those who were then in their 2nd period. In other words, the free-market equilibrium would not result in the social optimum, and therefore the free market should not be relied on. Instead, a social contract that extends even to the as-yet-unborn should create the social optimum, as reflected by the above example and a 0% interest rate. According to this social contract, one should surrender some of the goods produced in the active period and hand it over to the older generation at a 0% rate of interest, then receive it back in old age (but not from the same generation); this is to continue through an infinite sequence of generations. This is organised by social insurance (SI), and thus the standard of living is improved for all.

Samuelson considers production to be stationary throughout (i.e. 1 person produces 1 unit of chocolate in 1 period), but he also looks at a scenario of population increase, in which the population grows at a rate of m , multiplying by $(1+m)$ in each period. He finds that rather than at a rate of interest of $i=0\%$, a periodic interest rate of $i=m$ ensures the social optimum under such conditions (i.e. the pension received in old age is equal to the contributions paid to SI previously, adjusted by this interest rate). He calls this a “biological” interest rate.

Samuelson posits that if everyone insisted on *quid pro quo*, everyone would be worse off than the social optimum (the biological interest rate, in the absence of SI) until the end of time. This makes it expedient for mankind to conclude a Hobbes-Rousseau type of social contract, in which the young are given a guarantee of support in their old age if they support the older generation today; this applies to the as-yet-unborn as well. This makes the social optimum achievable in a single lifetime.

At the end of his paper, Samuelson points out that while such a social contract is essential in a moneyless economy to achieve a social optimum, it may be replaced by financial savings, making the situation different in a monetised economy.

In his analysis, Samuelson also makes the following assumptions, which are discussed below. This social contract has existed since time immemorial in economies, both in static populations (which, needless to say, represents a special case of a growing population, in which $m=0$) and growing populations; and m has also been unchanged for an infinite period of time. He does mention that there have been times in the past when this was different, as children supported their aged parents; this, however, has gone out of fashion since then. Nevertheless, he considers the old system and the new one to be similar in that advanced social insurance may also cause a bias towards a growing population, similarly to what is observed among farmers and groups with close family ties, where children are desired as a means for support in old age.

He assumes that m is basically higher than 0, but mentions that it may be negative as well, even citing some examples (namely, Ireland and Sweden).

Another simplification is that he assumes that people in the economy are entirely uniform: there are no genders or couples (he does not even mention this aspect, and he talks only about individuals). Furthermore, the paper does not even touch on the problem of raising children. Nobody dies prematurely, everybody lives for exactly three periods (or, rather, four, if the period of childhood, which is left out of his analysis, is included), these periods fit together exactly, and the life of the population is synchronised with these periods.

His model uses the following notation:

t	a one-period duration, which may take positive or negative integers as values
C_1, C_2, C_3	the consumption in the three periods
i_t	the interest rate
$R_t = \frac{1}{1+i_t}$	the discount rate
B_t	the number of cohorts
S_t	savings

The budget equation for one person is:

$$C_1 + C_2 \cdot R_t + C_3 \cdot R_t \cdot R_{t+1} = 1 + 1 \cdot R_t + 0 \cdot R_t \cdot R_{t+1}. \quad (1)$$

All this says is that the discounted consumption of an entire life must be equal to its discounted production.

The demand functions is thus:

$$C_i = C_i(R_t, R_{t+1}) \quad (i=1,2,3). \quad (2)$$

Samuelson seeks the equilibrium values of ... R_t, R_{t+1}, \dots that will clear the competitive markets in which the current and the future goods will be exchanged. He also defines the utility function (U) for this.

In addition, he defines “net” demand, which is the algebraic difference between a person’s consumption and output. Net demand in this sense is the negative equivalent of what people normally call “savings” (S). This is how he defines it, in accordance with capital theory:

$$S_1 = S_1(R_t, R_{t+1}) = 1 - C_1(R_t, R_{t+1}), \quad (3)$$

$$S_2 = S_2(R_t, R_{t+1}) = 1 - C_2(R_t, R_{t+1}),$$

$$S_3 = S_3(R_t, R_{t+1}) = 0 - C_3(R_t, R_{t+1}).$$

In old age, S_3 is negative, corresponding to the positive savings made in young age, so that the budget is:

$$S_1 + S_2 \cdot R_t + S_3 \cdot R_t \cdot R_{t+1} = 0. \quad (4)$$

In a stationary population, for all values of t ; in an increasing or decreasing population, $B_t = B \cdot (1+m)^t$.

First, it is necessary to stress that the above description disregards childhood consumption. The following will attempt to resolve this shortcoming to see what happens initially in a static population model like Samuelson’s and then in a growing population model. To recap, Samuelson essentially disregards children and childhood. Children are born *per se*, not as the result of an economic decision; they cost nothing (their consumption is part of the adult’s consumption, but does not reduce it) and they consume nothing. The first stage of life that Samuelson analyses is the active period starting at age 20. While children are not included in the economic exchange, they are part of the social contract, which essentially starts applying to them at the age of twenty. So what happens if this is not the case, but children also consume?

The rest of the paper does not look into what Samuelson devoted so much effort to investigating – namely, what conditions allow a market-based exchange among selfish individuals over a long period of time – because it is not difficult to understand the conclusion in his paper that this is essentially impossible and a social contract is required. Instead, it focuses on using the equation system to investigate possible templates for this social contract.

3 The case of the Stationary population

3.1 The modified model and a simple numerical example

On the basis of the above, an additional period “0” is introduced to precede the three periods used by Samuelson. The resulting four periods represent roughly the following age brackets: 0 to 20 years, 21 to 40, 41 to 60, 61 to 80.¹ There are two active and two inactive periods. Maintaining Samuelson’s other assumptions (such as, for instance, that everyone produces 1 unit in each of their active periods), this modifies budget equation (1) as follows: (1’)

$$C_0 + C_1 \cdot R_t + C_2 \cdot R_t \cdot R_{(t+1)} + C_3 \cdot R_t \cdot R_{(t+1)} \cdot R_{(t+2)} = 0 + 1 \cdot R_t + 1 \cdot R_t \cdot R_{(t+1)} + 0 \cdot R_t \cdot R_{(t+1)} \cdot R_{(t+2)} \quad (1')$$

The savings (3) will appear the same in the first approach, with only a further equation being added; this is S_0 , which will evidently take a negative value, C_1 :

$$S_0 = 0 - C_1, \quad (3')$$

$$S_1 = 1 - C_2,$$

$$S_2 = 1 - C_3,$$

$$S_3 = 0 - C_4.$$

¹ Incidentally, Samuelson’s two active periods lasted from age 20 to 65, while the third period (of retirement) was 15 years long. This is an inconsistency, because he holds one of the two active periods, totalling 45 years of total length, to be equivalent to the inactive period; the active period is actually one and a half times longer). However, this issue is not addressed specifically here. Instead, a more consistent period division is instead proposed, as seen above. This is not especially relevant in any case, but merely a tool to help with visualisation.

And this is appended to the budget identity (4) as well, as follows: (4')

$$S_0 + S_1 \cdot R_t + S_2 \cdot R_t \cdot R_{t+1} + S_3 \cdot R_t \cdot R_{t+1} \cdot R_{t+2} = 0. \quad (4')$$

In the stationary population, everyone has exactly one child (which is the same as each couple having two children).² It is assumed here, similarly to Samuelson, that everyone has the same consumption. Also unchanged is the assumption that active persons are involved in production in two periods, producing 1 in each of those. It already follows from these assumptions that everyone consumes the 2 units of chocolate produced in their active periods over the course of their entire life. In other words, it is true that:

$$C_0 + C_1 + C_2 + C_3 = 2 \quad (5')$$

or that the net saving over the course of life is 0, in line with.

$$S_0 + S_1 + S_2 + S_3 = 0 \quad (6')$$

This modified equation system has $R=1$ (i.e. $i=0$) as a solution, just as in Samuelson's. In this case, (1') essentially transforms into (5') and (4') into (6').

Providing a mathematical example with a possible concrete solution will allow for interpretation of the results.

It is obvious to assume that consumption will be smooth over the course of the entire lifetime:

$$C_0 = C_1 = C_2 = C_3 = 0,5.$$

The best way to envision this is to assume that an active person in period 1 has a child, shares with them the 1 unit of chocolate produced in the first period, and raises the child before the end of the period. In the second period, the active person shares the 1 unit of chocolate they produce with their now inactive, retired parent. According to the savings equations (3'):

$$S_0 = 0 - C_1 = -0,5,$$

$$S_1 = 1 - C_2 = 0,5,$$

$$S_2 = 1 - C_3 = 0,5,$$

$$S_3 = 0 - C_4 = -0,5.$$

² For the sake of simplicity, it is assumed that there is one boy and one girl. The father raises the boy and shares his output with him and the mother does the same with the girl (i.e. each child belongs to one parent and each parent to one child). It is necessary to make very generic assumptions that are nevertheless more detailed than Samuelson's, since we have also included childhood in our scope of analysis.

3.2 Interpreting the solution – a different scenario, a different kind of social contract

The above distribution is not suitable for interpreting this solution in the way that Samuelson interpreted his (i.e. without taking period 0 into account). The social contract here is not across an infinite number of generations, where each gives something to the previous generation and receives it back from the next one indefinitely; instead, exchanges take place here in a closed threefold set of generations of child-parent-grandparent: the parent gives to the child just as they had received from their parent in the past (and so on). This creates debt (negative savings) for the child vis-à-vis the parent, which the child returns in their 2nd active period. An active person in period 1 invests 0.5 savings in raising their child, in effect lending it to the child, whereas an active person in period 2 uses their savings for repaying the debt they incurred in period 0. Even though the generations overlap, each transaction will come to an end within three generations: the child is a recipient in their childhood and repays upon having raised their own child.

This is a sort of social contract, of course, but entirely different from Samuelson's: the pension is the repayment made by children to the parents for the costs of raising them. This is nothing else than the system that Samuelson deemed as having gone out of fashion. Is it indeed so? Does inclusion of the state in the process, to collect contributions and distribute pensions, change the old method substantively? The equations suggest that no, it will not, that it is merely a technical difference that the equations do not reveal. Accordingly, the scenario of creating SI differs from what Samuelson says. According to him, it had gone out of fashion for children to support their parents (as had been the case in the agrarian societies of the past, where it was easy because multiple generations cohabited and children reciprocated the care they had received from their parents) and, therefore, it was necessary to enter into a new social contract. In this social contract, all transfers would be strictly "forward" (i.e. give to the preceding generation and receive from the next generation), which is facilitated by organising SI.

In this scenario, subsequent to the collapse of agrarian societies, different generations no longer cohabit and supporting parents is now more difficult. Many have, in fact, dispensed with this obligation as it has become easy to do, compared to when generations cohabited in a community and observation of the norms, such as the support a child gave to their aged parents, was monitored. The state had to step in and organise the transfer from children to parents (i.e. repayment by the children of the costs of raising them, which now took the form of pensions). In essence, however, nothing had changed. This scenario could therefore be called the old scenario and Samuelson's the new scenario; however, I have preferred to name them after the generations participating in them. Accordingly, Samuelson's scenario is an Active-Inactive (in short, AI) scenario, since the children (who are also inactive) do not participate in it. The traditional pension system is an Inactive-Active-Inactive (in short, IAI) scenario. The assertion here, essentially, is that there has been

no change, and there is no need for a new scenario as the old one worked. The modern pension system scenario could be an IAI scenario as well.

The economy proposed here, therefore, deviates from Samuelson's in that the participants do not even attempt to "do business" with the other (younger) active persons, but do so with the children – right from the start. This is no longer possible directly, as it was in the past, but involves the state organising it.³

The above mathematical example is overly specific, but the interpretation is obviously relevant for all other solutions where

$$C_0 = C_3,$$

because the same logic is at work. Accordingly, if the older generation consumes exactly as much as they gave their children in the past, it becomes entirely clear that this is a matter of returning something.

Naturally, $C_0 = C_3$ and $C_1 = C_2$ may take a wide range of relative values, but the most relevant assumption is probably that $C_0 < C_1$, (i.e. children and old people consume less chocolate than active persons). It is easy to find an ideology to support this in simple economic terms: a child is physically smaller than an adult and thus consumes less; active persons must work and eat more; and old people do not work and, as a result, do not need to consume as much.

The interpretation does not change even if $C_0 > C_3$. In this case, raising children simply returns a negative rate of interest; after all, this is required for security in old age. It is still the costs of their care that the children will repay – just with a negative rate of interest. The situation becomes slightly more problematic when $C_0 > C_3$.

3.3 A slightly more complicated mathematical example and its two possible interpretations

The following practical objection may be raised: a child consumes less than an adult, but adults seek to smooth their consumption over their whole adult life. In this case:

$$C_0 < C_1 = C_2 = C_3.$$

³ The above model does not differ from Samuelson's in the sense that they both greatly simplify the actual situation; in Samuelson, this was later resolved by practice grounded in his theory, whereas there remains here the responsibility of describing the system in greater detail. The simplification lies mainly in that both of us disregard the fact that lifetimes are not as standardised as in this model, which necessitates an actual system to have a variety of methods for risk equalisation.

To simplify matters, specific numbers can be used:

$$C_0=0,2,$$

$$C_1=C_2=C_3=0,6.$$

According to the above, the distribution is: in period 1, the active person consumes 0.6 of their 1 unit of chocolate output, gives 0.2 to their child and 0.2 to their grandparent. In period 2, the active person consumes 0.6 of their 1 unit of chocolate output and gives 0.4 to their parent, supplementing the 0.2 given by their child to add up to 0.6; accordingly, the parent/grandparent in period 3 is able to consume 0.6.

3.3.1 Two kinds of explanations

The 0.6 consumption by the inactive person in period 3 may be explained in two ways:

Explanation 1: the childcare investment of 0.2 made by the young parent in period 1 has returned a 0.6 profit in period 3 (a yield of 200% over two periods).

Explanation 2: there were two sources for the 0.6 consumption in period 3.

- a. The pensioner received the 0.2 unchanged (for the sake of simplicity, assume that they received it from their child, the active person in period 2, who also gave them 0.4), and
- b. received 0.2 from two subsequent generations, whose transfer to the generation before them can be interpreted as their way of smoothing their consumption over their entire adult lifetimes. They do not, of course, do this by way of market exchange (which Samuelson proved to be impossible); instead, they use the same SI mechanism invented (according to the IAI scenario) to make sure that children repay their parents the costs of raising them.

Two explanations, two different scenarios, two different social contracts! The first explanation is our mutually embedded, three-generation series of social contracts above, whereas the second one is a combination of this and Samuelson's social contract of infinite generations.

3.3.2 Selecting from the explanations

Which explanation is true? One may notice right away that the new, three-generation series of mutually embedded social contracts is present by all means, whereas Samuelson's is optional. It will be clearly absent if consumption in old age is not greater (let alone smaller!) than consumption in childhood (i.e. there is no consumption smoothing in adulthood). It seems, in fact, that real pension systems tend not to provide pension amounts that would lead to this assumption.

Nevertheless, if they do provide it, the question arises whether we are facing scenario a) a plain explanation, or b) a combined one.

In a real pension system (which is, of course, much more complex than the model here), the best way to decide this question may be the practical one of comparing the benefits from the specific pension system with the costs of childcare, with due reference to changes in productivity as well.⁴ If either provides more than the other, then it can include an income-smoothing component as well. We must remember, however, that in this empirical analysis the costs of parenting comprise not only the transfer of physical goods (“chocolate”), as in our model, but also a considerable investment of time, which may not be a direct cash expenditure but cause the loss of opportunities to earn income (typically among mothers). If the benefit were lower than this, then an increase would be necessary.

3.3.3 SI is not the best means for income smoothing

Aside from how to resolve the above question in practice, there is another matter that needs an answer: is it expedient to use the pension system for income smoothing in old age or would a different method be advisable? There are several reasons why the answer is no:

1. No, because the pension system is intended for a different purpose. The pension system protects primarily from extreme poverty in old age, but is not tasked with ensuring the accustomed level of welfare. Samuelson does not consider this a motive for the pension system, which he finds simply as protection against death from starvation.
2. In Samuelson’s model, goods cannot be stored. Therefore, the pension system cannot be resolved through the stockpiling of physical goods, necessitating a social contract between generations. He mentions at the end of his paper, however, that this – purely this formulation – applies only to moneyless economies, and as money is introduced, an entire different pension system becomes possible, one that is based on savings. In today’s world, there are practically no moneyless economies; therefore, monetary savings are the best means for the smoothing of consumption – if it is needed in the first place.
3. In the modern environment where the old family units have dissolved, a law may be needed to require that the costs of child-rearing be repaid. Consumption smoothing, however, is a private matter, and there are no sound reasons for making it mandatory by elevating it to the level of a national system.
4. Finally, the terms of the modern pension systems themselves prevent consumption smoothing in the active periods. The way that they function today is not very conducive to smoothing. This can be examined in greater detail below.

⁴ This is not discussed in this paper. It should be noted, however, that Aaron (1966) happened to use exactly the same factor, rightfully, to adjust Samuelson’s model.

3.3.4 The SI and consumption smoothing

The above mathematical examples have adhered to the following principle: an active person in period 1 consumes most of the goods they produce and also gives goods to their child. If anything remains over and above this amount, they give it to the inactive person in period 3, who will receive more than this from the active person in period 2, as the latter no longer has any child-rearing expenditures. By contrast, the SI logic assumes that all active persons pay the same contributions (in percentage terms, but this coincides with the absolute value in our model). The expenditures can be seen as follows.

The active person in period 1 spends C_0 on child-rearing and pays $\frac{C_1}{2}$ in contributions, so they will have $c_1 = 1 - c_0 - \frac{C_1}{2}$ left.

An active person in period 2 will have more left, however: $c_2 = 1 - c_0 - \frac{C_1}{2}$, since they no longer need to pay for child-rearing, only the SI contributions. Accordingly, even if the SI were to work towards consumption smoothing, it would need to do so first in the period of contribution payment, by applying a lower rate to those raising children than those no longer raising any. In other words: it would make sense to insert a chronological demarcation between child-rearing (=investment in the next generation to create my own pension) and the repayment of the costs of raising us. This would also achieve a kind of (approximate) “generational fit”, as the pensioners would receive the costs of child-rearing back from the very generation that they raised.

4 The case of the non-stationary population

4.1 Samuelson’s mathematical example

Analysing even the case of a stationary population puts the pension system in a new perspective. But while it suggests a new explanation and different kind of scenario, Samuelson’s assertion that we should work with an interest rate of around 0% does not need to be modified. Admittedly, it should be projected on child-rearing costs rather than the SI contribution.

In the IAI scenario introduced here, to what extent does the interest rate m apply in the event of a growing population?

Samuelson’s main assertion concerning a non-stationary population increasing (or decreasing) by a factor of $1+m$ (per period) is that the pension may also increase: namely,

the contributions are adjusted by the m rate of interest per period to receive the pension figure. Accordingly, our contributions have a (“biological”) interest rate of $i=m$.

To consider Samuelson’s specific example (and carry the calculation somewhat further) in order to understand how it works in his understanding, he does not include a period 0 and assumes, for the sake of illustration, that $c_1=c_2=\frac{2}{3}$ (i.e. the intention is to smooth out the consumption of the 1 unit produced in the active period). In a stationary case, the SI contributions of $1/3$ each by two active persons will result in $2/3$ consumption in old age. Assuming now (also for the sake of simplicity) that $m=100\%$ (i.e. the population doubles in each period, everyone has two children, and every couple has four) and that this has been the case for a long time, there are twice as many persons in period 2 than in period 1, and four times as many in period 3. Accordingly, every pensioner in period 3 will have 6 active contributors. If each of these pay $1/3$ of a contribution share, then the pensioner will receive a total of 2 units, which represents 100% interest per period on their contribution payments, since if $m=1$, then $1+m=2$, and

$$\frac{1}{3} \times 2^2 + \frac{1}{3} \times 2^1 = 2$$

4.2 Samuelson’s example according to the IAI scenario

To match the IAI scenario, Samuelson’s example undergoes the following modifications:

- The children are also involved in consumption (e.g. $1/3$). This is also included in the 2 units of chocolate that the person will have to produce, who will thus be able to consume a total of $5/3$ in adulthood.
- The pension is based on children returning the costs of their care (i.e. the parents enter into contracts with their children).
- Otherwise this scenario also assumes two children to be raised by each person (four per couple), as in the example above, and a continuation of this process over multiple generations. In other words, everyone has “half” a parent to support (i.e. every parent is supported by two out of their four children).

In this instance, the above example is modified as follows:

- In period 0, a person receives $1/3$ of a unit from one of their parents.
- In period 1, the same person consumes $1/3$ of a unit and gives $2/3$ to their two children.
- In period 2, the person consumes $2/3$ and gives $1/3$ to their parent, who thus receives a total of $2/3$ from their two children.

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- In period 3, the person consumes $\frac{2}{3}$, received from their two children.

In this way, one consumes a total of 2 units and earns a total of 2 units.

There are several differences between this example and Samuelson's example:

- In this model, the person consumes only $\frac{1}{3}$ of a unit in period 1, spending the rest on child-rearing. Samuelson does not recognise child-rearing costs, and thus consumption is $\frac{2}{3}$.
- In period 1, pension contributions are not paid. Instead, the $\frac{1}{3}$ for this is spent on child-rearing.
- Period 2 is the same here as in Samuelson.
- Period 3 is the same as in Samuelson's stationary model.

The main difference is that there is no "biological" interest on the contribution in the IAI scenario, nor is there any on child-rearing: one gets back what one has spent on the child. Compared to the stationary situation in the IAI scenario, the pension is twice as high, since there are two children; this has a price, however, as consumption in young age is lower by the amount spent on child-rearing.

In other words, there is a multiplier of 2 for the pension compared to the stationary scenario, but this does not imply periodic compound interest, merely the fact that the pension will be twice as high if there are two children. But Samuelson's "yield" has evaporated and disappeared. Indeed, it is nowhere to be found! So where did Samuelson get it from? The situation is very simple: the main difference between the two examples is that Samuelson rechanneled some of the child-rearing costs of the populous first generation to the pensioners! In other words, he set the benefits too high for the older generation, directing it away from child-rearing!

Is this expedient? Our answer will depend on which scenario concluded from Samuelson's model (slightly modified, made to some extent more realistic) is deemed to be correct: the AI (Samuelson's) scenario or the IAI scenario, outlined above.

5 The two scenarios

To summarise therefore: Samuelson's model can be interpreted in at least two ways not just one, as a sort of social contract. The two interpretations describe the antecedents as different scenarios, of which I provide a brief summary here:

5.1 The AI scenario

According to Samuelson's scenario, called the AI scenario, people in the agrarian societies of the past used to achieve security for their old age through cohabitation of the generations, as active persons supported their old parents, who lived with them, which is why it was important to raise children ("traditional pension system"). This has now gone out of fashion (presumably because generations no longer live together), and therefore a new kind of social contract was entered into and social insurance was organised to ensure compliance ("modern pension system"). According to this, active adults give up some of their earnings for the benefit of elderly inactive persons, in return for which they can expect to be supported by the future generation on the basis of the social contract that applies to children and the as-yet-unborn as well, and so on. In this way, payment of contributions creates the grounds for eligibility for a pension.

The whole social contract is predicated on having a sufficiently large next generation (i.e. a system of pensions payable pursuant to a social contract demands a sufficient number of children, just as the earlier solution of children supporting their own parents did); in contrast to that, however, the system does not incorporate children and child-rearing explicitly in any way.

5.2 The IAI scenario

There is, however, a different way to tell the story of how the modern pension system of SI evolved. According to this, the traditional and the modern pension systems are essentially identical, and there is no new (AI) scenario, as the old one (IAI) remains in effect: it is the children and grandchildren who support the parents and grandparents today as well. Albeit there have been a number of technical changes, since the old arrangement of solving all this within a shared household is no longer feasible, as shared households have dissolved. Moreover, support in the traditional pension system was not predicated fundamentally on money transfers, whereas today, when the generations live apart, money is the only way of support. The new conditions are also different from the old ones in the sense that it is now easy for children to get out of their obligations, making it necessary for the state to set up social insurance to organise the (now strictly monetary) transfer from children and grandchildren to parents and grandparents. Accordingly, the social contract is not about active persons supporting the elderly, in return for which the active persons of the next generation will support those active today in their old age. What is happening is, in fact, the same as in the past: young people pay old people back the costs of raising them (perhaps with interest, perhaps only in part) and their children will do the same for them.

Pensions, therefore, remain nothing other than the recovery of investment in children; pension contributions are not a kind of “advance saving” for a future pension, but the repayment of child-rearing costs.

This means that, in accordance with the IAI scenario, the traditional and the modern pension systems differ from one another not in essence, in the nature of the social contract, but technically; after all, both are predicated on the same premise: that investment in children is the basis for future pensions.

Although the IAI scenario does not posit a substantive difference, but only technical differences between the modern and the traditional pension systems, these technical differences are huge. Admittedly, they concern implementation rather than essence. In the traditional pension system, investments in children and their returns were strictly on an individual basis.⁵ When the pension system is organised by the state in the modern system, the state spreads risk to a certain extent: it makes the sustenance of the old person independent of whether specifically their child is willing to support them, making the contributions (or the repayment of child-rearing costs) mandatory. The state also makes it irrespective of how well the child is doing on the labour market and how much money they have, if any, to repay the costs of raising them. An insurance component is also included in the system, as the risk of a long lifetime is spread over time, which exempts children from the risk of their parents living too long and exempts parents from the risk of their children dying before them.

The costs of child-rearing (which are the basis for the pension) are estimated by proxy (namely, the income of the parent), which is taken into account when calculating the pension.

5.3 The differences and the similarities between the two scenarios

The two scenarios explain the different characteristics of modern pension systems differently, and various diverging policy conclusions may be drawn from them:

- In the AI scenario, the pension is a reward for contributions previously paid to finance the sustenance of the elderly (i.e. the contribution is an investment), whereas in the IAI scenario it is the “proceeds” from child-rearing (i.e. the investment was the child-rearing). The pension is the proceeds of two different investments in the two different scenarios.

⁵ The adjective “individual” should not be interpreted in the modern sense; this took place within the joint family.

- Contributions have different statuses in the two scenarios: they are an investment generating a yield in one and a repayment of debt in the other. The latter, naturally, does not give grounds for any proceeds; on the contrary, interest may be paid on it. According to the AI scenario, the payment of contributions is, in effect, “forced saving” (and as such, it is essentially similar to the forced saving in mandatory, but funded pension systems), whereas in the IAI scenario it is the “forced” repayment of child-rearing costs.
- The status of child-rearing is also dissimilar in the two scenarios: it is a private hobby in the former, one that has nothing to do with pensions (although it ultimately underlies the pension system, albeit not explicitly), whereas in the latter it is a central component of the pension system.
- In the AI scenario, the “internal rate of return” of the pay-as-you-go pension system is calculated by comparing the contributions paid and the pensions received, whereas in the IAI scenario it is calculated by contrasting the child-rearing costs with the pensions.
- The fact that contribution payments are investments in the AI scenario also implies that there is government debt underlying them implicitly (“implicit government debt”), as the pension is “due” in return for the contributions paid. In the IAI scenario, there is no implicit government debt within the pension system, since the payment of contributions represents a repayment of debt by the individual; at the same time, this is also the basis for the pension.

Otherwise, the two scenarios describe very similar pension systems. In both, the state uses the SI to force active persons to give up some of their income and transfer it to the elderly. Neither scenario assumes the existence of reserves in the pension system (or only short-term ones), but while this may be considered a shortcoming in one, it is self-evident in the other. The main difference lies in the basis for determining the pension: in the AI scenario, the basis is the contributions paid; in the IAI scenario, the basis is the children raised, who will contribute.

5.4 Which scenario fits today’s pension systems better?

Modern pay-as-you-go pension systems were created solely on the basis of the AI scenario, which serves as their official philosophy. It is possible to argue, however, that the modern pension systems created right at the start as pay-as-you-go systems are described better with the IAI scenario. The quintessence of this, after all, is the problem of a lack of care for the elderly (in other words, the refusal by young people to honour their earlier obligation of supporting their aged parents, which could be interpreted as a repayment of the costs of raising them). It was, therefore, necessary for the state to exert pressure. It did not do

this on an individual-by-individual basis,⁶ however, but created a system incorporating certain rational (and one less rational) risk-sharing elements and a degree of redistribution, thereby increasing the security of society.

It is important to mention, however, that pay-as-you-go systems were created in one of two ways (see studies by György Németh on the subject, such as Németh, 2009): either by creating them as such right from the start, as in the USA, or due to the pension system “plummeting” into this situation from an earlier funded system when underlying capital disappeared, but entitlements were still recognised (and the system subsequently proceeded to operate as pay-as-you-go). It may be argued in the latter case that classifying this as forced saving continues to be justified, but we must note that such funded pension systems extended only to a small part of the population when their capital was lost and, once they were extended to other groups, the same logic as in the USA started to predominate.

The IAI scenario sheds light on who benefited from the introduction of the pay-as-you-go pension system. In both scenarios, the use of various risk equalisations was beneficial for most stakeholders. Where the two scenarios differ is that, in the AI scenario, those who received their pension after a short period of contribution payments following the introduction of the system benefited in particular, since they accessed the pension “undeservedly”. According to the AI scenario, there is implicit government debt rather than actual capital underlying the system because the contributions of the first pensioners are still missing.

By contrast, the IAI scenario interprets this differently: children dispensed with their obligation of repaying the costs of their care (i.e. by supporting their aged parents), and the state later forced them to do so by setting up the modern SI and levying contributions. In this scenario, the elderly who received their pensions then did so “deservedly”, and the ones to benefit the most were those who previously had neither paid contributions nor supported their parents, as well as those who received a pension without actually raising children.

⁶ An example of the latter solution may be found in China, where the constitution declares that children have an obligation to support their parents. This is a clear sign of the survival of the traditional pension system. Its philosophy is the same as that of the IAI scenario, but the specifics of the solution do not correspond to those of a modern pension system.

6 Conclusions and lessons

6.1 The internal contradictions of modern pension systems

In light of the above, it is an inherent contradiction of modern pension systems that they are predicated on children and their potential as contributors, yet they do not reward child-rearing; on the contrary, they disincentivise it.⁷ This is because the modern pension systems were created with the inclusion of an additional risk equalisation to the aforementioned ones: namely, they made the pension irrespective of whether a couple managed to have children or not. It seems that they assumed that everyone wanted children, but only some succeeded, and they also assumed that the same applied to the success or failure to start a family at all. If we assume that having a family is an objective for everyone and only unlucky people will not have children in their family, then a kind of solidaristic element of redistribution exists within the system. If, however, these assumptions are not true, then it is an undesirable element of redistribution in the system, which causes a free-rider problem and requires correction.

The situation is even worse: in fact, this arrangement rewards free riders, as persons who do not have children are able to pursue formal earning activities longer, which then results in higher pensions. Accordingly, it is economically rational not to have children once a modern SI has been established (in contrast to the traditional pension system), which in turn jeopardises subsequent pensions, as there will be no contributions and thus no pensions if there are no children.

It has been observed that pay-as-you-go pension systems reduce fertility. The IAI scenario tells this story differently. According to that scenario, it is not the pay-as-you-go system itself that reduces fertility, but a single component of it: namely, that the system is organised in a manner that “socialises” children. In other words, the benefits from raising children are divided equally among those who contribute and those who do not contribute to creating them, thus making free rides rational (which, in this particular case, means saving on the costs of child-rearing).

The new explanation also maintains that the pay-as-you-go pension system is needed, but should be the institutional form of repaying the costs of child-rearing, since the old, direct arrangement is no longer feasible as the division of labour has changed.

⁷ This overall judgement is not changed, but only slightly modified by the fact that a large number of pension systems, including in Hungary, incorporate components that reward child-rearing (to a degree). In the Hungarian system, such a feature is the inclusion of the time period spent on childcare in the total period of service.

6.2 Potential transition between the two scenarios

Due to the aforementioned structural error, an internal contradiction has widely pushed the modern pension systems into a crisis, as almost nowhere do they have a sufficient number of children to finance them. Their revision is, therefore, on the agenda. A logical path for revision would entail the discarding of the AI scenario and drawing conclusions from the IAI scenario. One might even say that developments in pension systems have made it impossible to use the AI scenario for explaining the implicit social contract, forcing the use of the IAI scenario and the changes it implies. The most important of these changes is that contribution payments as such will not create eligibility for a pension, only successful child-rearing will. A child is raised successfully if they become a contributor.

Naturally, parents do not bear 100% of the costs of child-rearing today. For instance, most of the costs of schooling and healthcare for children are socialised (i.e. paid for by the community of taxpayers). Also, persons who raise children receive a variety of transfers (e.g. family allowance, tax relief), which also come from the taxpayers. Accordingly, childless persons do contribute to raising the next generation, to some extent, through the taxes they pay; therefore, they should also receive some of its benefits. However, that is not equal to the amount due to those with children (persons with children are understood here as those who are raising contributor children or are financing their child-rearing, regardless whether they are their biological children or not). The way to formulate a reformed pension system is, therefore, to declare that those who successfully raise one contributor per person (i.e. two per couple) will receive approximately as much pension as the average pensioner does today. The childless will receive much less and those with several children will receive more.

Also, it must be made clear to all that there are two options regarding pensions:

1. Either one spends some percentage of earnings and free time on child-rearing, in which case the full pension in the pay-as-you-go pension system is received, or
2. One saves on the costs of child-rearing and receives a much lower pension, but one's own savings and the annuities purchased with them may be added (and the source of one's own savings will come from the child-rearing costs that were not spent).

Naturally, it is possible for individuals to make this choice only up to a certain age. Accordingly, the transition to the pension system of the IAI scenario must be phased: it should be introduced comprehensively only to those who are still able to adapt to this situation. Others, who have lived their lives according to the old philosophy (of Samuelson), should receive their pensions on that basis.

6.3 About NDC

One popular way of reforming the pension system today may involve a transition to a system of Notional Defined Contribution (NDC). It is, therefore, necessary to comment on NDC in light of the IAI scenario, especially as this author has proposed reforming the Hungarian pension system on an NDC basis (Banyár-Mészáros, 2003).

Clearly, NDC belongs fundamentally to the AI scenario and represents its most logical version, one that is rationalised to the utmost! Its starting point is that the pension system can be saved primarily by stimulating contribution payments, and that this presupposes accurate and correct accounts. Its other recognition is that the monthly pension level is not a good measure of pensions, since a person retiring young will receive much more in total than someone retiring at a later age, even if their pensions are level. The total expected pension figure should, therefore, be used as guidance; equal pension means equal expected total pension. For this reason, pensions must be calculated in accordance with the rules of annuity calculations.

The IAI scenario does not differ from this at two important points: it also considers contributor discipline to be necessary and it prefers comparing pensions at the expected total pension level rather than at the monthly pension level. The IAI scenario also deems correct accounts to be important, although it interprets this somewhat differently: contributions as repayment of child-rearing costs must be paid correctly, and contributions to child-rearing must be correctly accounted for as well. In terms of the latter, there are two types: the payment of taxes and the actual raising of children.

It is difficult to account for taxes paid, of course, but there are proxies available. The individual pension contribution may be a good proxy – provided that its level is set properly. Thus, the individual account included in the technical solution for NDC may serve as a basic component of a pension system based on the IAI scenario, but only the amount on the account must be increased with the estimated costs of child-rearing. Individual accounts will also be needed for the purpose of comparison of the expected total pension level, as well as to be able to reward those retiring later with a proper rate of increase in their pensions.

The IAI scenario and behavioural economics

The IAI scenario highlights the AI scenario's mistaken interpretation of affairs. This can be explained on the basis of behavioural economics, namely as a form of WYSIATI ("what you see is all there is") (for a definition of WYSIATI, see Kahneman, 2013). In essence, the AI scenario did not take into account the role of child-rearing. This may be due to the fact that child-rearing usually does not involve a financial transfer, but a sacrifice of time;

moreover, the process does not go through the system of state redistribution. By contrast, pensions are channelled through that system in their entirety, and therefore it seems all there is (WYSIATI). All that one needs to do is compare these transfers with one another.⁸

7 The afterlife of Samuelson's paper, attempts to supersede the AI scenario

as one of the main points of reference since 1958, Samuelson's paper has been used to explain the structure of the pay-as-you-go pension systems and considered as key in the laying of the economic foundations for the pension system (see Robin Blackburn's thick tome of 550 pages on the history of pensions, in which he describes Samuelson's paper in this way and refers to it many times; Blackburn, 2002). The pay-as-you-go pension systems had, of course, been created previously, but experts until then had nursed a fear that this was in fact a Ponzi scheme that would collapse one day. Samuelson's paper dispelled this fear, claiming that the system was predicated on solid theoretical foundations (Blackburn, 2002).

Samuelson's paper influenced pension modelling in another way as well. After his model, it became generally accepted to consider the active age as the start of the human lifetime, completely disregarding the period of growing up and the costs associated with it (in Hungarian pension theory, Simonovits provides one example of this approach; see Simonovits, 2002).

Another paper that is frequently quoted besides Samuelson's is a brief, three-page study by Henry Aaron. Therefore, reference is often made to the Samuelson-Aaron model or theory. Aaron extended Samuelson's model in the sense that, besides population growth (which Samuelson analysed), he also took into account increases in real wages, formulating the assertion that the pay-as-you-go system increases welfare compared to the funded systems if population growth plus real wage growth exceeds the rates of interest. This indicator was later referred to as the internal rate of return of the pay-as-you-go pension system.

It should be noted here that Samuelson's paper was not initially intended so much as a theoretical foundation for pension systems, but was instead a by-product of a debate, in which Samuelson wanted to prove that the market did not solve everything well and that a social contract and a set of social institutions was necessary from time to time.⁹

⁸ Róbert Gál drew my attention to this.

⁹ I owe thanks to György Németh for pointing this out to me.

This article can be seen as proof that Samuelson would have achieved his original objective with his paper, even if he had presented this scenario for establishing social insurance; in truth, he would not even have had to change his model much, only his argumentation.

Incidentally, the conclusion of this article that the pay-as-you-go pension systems could have been explained differently right from the start – and thus their structure could have been changed at an important point – is not to be found in the literature. By contrast, there is nothing new in proposing the explicit inclusion of child-rearing in the type of model Samuelson puts forward in his paper (namely, the overlapping generations (OLG) model); this would mean making it endogenous, in contrast with Samuelson, and linking child-rearing with pensions (with or without a model). A few approaches may be cited, without attempting completeness.¹⁰

Alessandro Cigno was one of the first to suggest that the pay-as-you-go pension system itself contributed to decreasing productivity in the countries that employed such a system (Cigno, 1991). András Gábos, Róbert Iván Gál and Gábor Kézdi tested the assumption, using Hungarian data (Gábos-Gál-Kézdi, 2009), and found that productivity is very strongly influenced by monetary family allowances, whereas the pension system has a strong impact that is nevertheless weaker than that of family allowances. In a kind of conclusion from this, Cigno subsequently analysed the possibility for relating child-rearing to pensions (Cigno-Werding, 2007), where the pension would depend on the number of children raised and their ability to pay contributions.

Hans-Werner Sinn does not question the AI scenario, but his proposal is nevertheless very similar to the one concluded here with the IAI scenario (Sinn, 2001). He believes that if a pension system had to be created from scratch today, it would be a funded system, but the transition would impose excessive burdens on the current active population. It would also entail an inequitable aspect of especially punishing those who are raising children, even though they are not the reason for the crisis of the pay-as-you-go pension systems. Therefore, he recommends a selective, transitional funded pension arrangement in which pensions of the childless are capitalised from their contributions, at least in the first phase.

Mária Augusztinovics was one of the first in the world to connect child-rearing to the pension system (Augusztinovics, 1993). She quotes Samuelson, but even as she accepts his scenario she considers capitalisation of the pension system as a desirable path. She also asserts, however, that this must be coupled with a “capitalisation” of child-rearing and that the two capitalisations should be combined so that pension capital is invested primarily in raising and educating the new generation. This would continue until adulthood, at which time this new generation would need to invest into itself (i.e. finance its own higher education); this would take the form of student loans, which would serve as funds

¹⁰ I owe thanks here to Róbert Gál, who pointed out several of the publications cited.

for the pension system to a degree. It is this idea that Edina Berlinger takes further in her paper (Berlinger, 2005).

Augusztinovics's message could be reformulated in general terms based on this paper by declaring that in actual fact there is only one sort of good pension system – namely, the funded system – and the pay-as-you-go system predicated on Samuelson's AI scenario is not good. Based on the IAI scenario, however, pay-as-you-go represents a special kind of capitalisation: investment in human capital, with the pension as returns on this investment. This idea, albeit in different formulations, also exists in the literature. Mason and Lee (2004) wrote in similar terms about the demographic dividend (a phenomenon that occurs when the number of births has just started to decline, the expected lifetime of the elderly is not yet increasing, and the average lifetime of the large present-day active generation is lengthening; this increases the proportion of active persons within the population, as a result of which the latter have less of an obligation than before to provide sustenance). The authors maintain that a second demographic dividend may be possible if the demographic dividend itself is used for capitalising the system; a special form of this is investing capital in educating the new generation. This helps avoid the population growth resulting from the demographic dividend from leading to an eventual aging crisis, which would certainly occur if pay-as-you-go (of the type proposed by Samuelson) were introduced.

Van Groezen, Leers and Meijdam created a pension model that incorporates child benefits as well (Groezen-Leers-Meijdam, 2003). They propose that the state, besides levying a pension contribution on active persons so that pensions can be paid to the elderly, should pay a subsidy proportionate to fertility from the taxes also levied on the active persons; these two items function as Siamese twins within the transfer system. The same line of thought is adopted in the current Hungarian literature by András Simonovits (2014), who considers the model of these three authors as a starting point for his own. He concludes that such transfers – and a similar pension, one that is dependent on productivity – will reduce social welfare, even as it increases productivity and social heterogeneity. This contribution is partly Simonovits's comment on the potential introduction in Hungary of pensions that are dependent on the number of children. Experts in this country have regularly re-tabled the debate on the subject (see Kovács, 2012 and Banyár-Mészáros, 2014).

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